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Entanglement Entropy in an Opened Antiferromagnetic Heisenberg Chain with Alternating Interaction

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Abstract The effects of alternating interaction on the bipartite entanglement are considered in an antiferromagnetic Heisenberg opened chain. Entropy is used to evaluate bipartite entanglement in the system. There are large oscillations between even and odd L-values entropy. The even (odd)-values entropy increases when the length of the subsystem increases. The method of density matrix renormalization-group is applied to obtain logarithmic behavior of entropy.

Keywords Entanglement entropy · Alternating interaction

1 Introduction

In the past few years, many-body system quantum entanglement has been paid much attention to, because entanglement is considered as the heart in quantum information and computation (QIC) [1, 2]. Many researches show that entanglement exists naturally in the spin chain when the temperature of system is at zero. In recent years, many researches focus on the entanglement measurement, such as concurrence [3], Neumann entropy [4] and perhaps others. The Neumann entropy is used to qualify the bipartite entanglement of pure states, which can quantify quantum phase transitions [5, 6]. The entanglement entropy in

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the antiferromagnetic Heisenberg XX chain and Ising chain are investigated [7]. In isotropic antiferromagnetic Heiseneberg model the universal form of entropy was predicted [8]

$$S_L = \frac{c}{3}\log_2 L + k, \tag{1}$$

where c is the central charge and k is a non-university constant. For an opened chain, the analogous formula $c/3$ must be replaced by $c/6$ hold for a part of length L in an infinite one-dimension system [9]. Recently, it is shown that a feeble central bound defect have a strong effect on entropy [10]. A weak transverse boundary magnetic field shows its strong influence on the entropy, though the entropy measures the mutual coupling of the two part of a system in wave function [11].

In condensed matters, a alternating Heisenberg antiferromagnic spin chain is generalized by the uniform Heisenberg chain simply, with the nearest-neighbor exchange constant replaced by two alternating values, such as $CuWO_4$ [12], $CuGeO_3$ [13] and $\alpha' - NaV_2O_5$ [14, 15]. It can be realized in experiment, and can not be exactly solved theoretically. Alternating interaction has shown its particularly effect in strongly correlated fermionic systems [16–19]. It is valuable to investigate the effect of alternation interaction on the bipartite entanglement in natural alternating spin chains.

In this paper, the entropy properties are considered in the alternating antiferromagnetic Heisenberg chain. In Sect. 2, the Hamiltonian of the system and entanglement measurement are presented. The entanglement entropy is studied by exact diagonalization and density matrix renormalization group (DMRG). At last, a brief discussion concludes the paper.

2 Hamiltonian and Entanglement Measurement

The alternating Heisenberg antiferromagnet is a simple Heisenberg spin chain, with the nearest-neighbor exchange two alternating interaction, the Hamiltonian of the system can be written as

$$H = J\sum_{i=1}^{N-1}[1-(-1)^i\delta]\vec{S}_i \cdot \vec{S}_{i+1}, \tag{2}$$

where $\vec{S}_i = \frac{1}{2}\vec{\sigma}_i$ is the i-th spin vector. The coupling exchange $J > 0$ correspond to the antiferromagnetic case. $J = 1$ is considered for simplification in the following paper. The number of spins of the lattice N is even and the open boundary condition (OBC) is assumed. The parameter δ denotes the alternation ratio of exchange interactions.

The entropy is selected as bipartite entanglement measure. It is defined as follow. Let $|Gs\rangle$ assign to the ground state of a chain of N qubits. A reduced density matrix of L contiguous qubits is written as

$$\rho_L = \mathrm{Tr}_{N-L}\,|Gs\rangle\langle Gs|. \tag{3}$$

The bipartite entanglement between the L contiguous qubits and the rest of system can be measure by entropy, which is described as

$$S_L = -\mathrm{Tr}(\rho_L \log_2 \rho_L). \tag{4}$$

A property of the entropy of a block of system is given

$$S_L = S_{N-L}, \tag{5}$$

because the spectrum of the reduced matrix ρ_L is the same as ρ_{N-L}. In the paper, we will only show S_L, $(L = 1, 2, \ldots, N/2)$ for simplicity.

3 Entanglement Entropy

The entanglement entropy S_L between contiguous L qubits and the remain $N - L$ qubits is calculated. For even L, the density matrices $\rho_{12}, \rho_{1234}, \ldots,$ are traced out in obtaining S_L. While for odd L, $\rho_{123}, \rho_{12345}, \ldots,$ are traced out. The two limit cases $\delta = \pm 1$ of the system are more interesting. For $\delta = -1$, the ground state becomes dimerized onto odd bonds. The even-values entropy $S_i = 0$ $(i = 2, 3, 4, \ldots)$ and odd-values entropy $S = 1$ $(i = 1, 3, 5, \ldots)$. For $\delta = 1$, the ground state becomes dimerized onto even bonds with one qubit in each two ends. The even-values entropy $S = 2$ and odd-values entropy $S = 1$. When $|\delta| \neq 1$, we diagonalize the hamiltonian accurately of a system contains $N = 20$ qubits. The entanglement entropy of ground state is obtained. There are large oscillations between even and odd L-values entropy in even-length chain. The odd and even L-values entropy are plotted in Fig. 1(a) and (b) respectively to avoid the oscillations. Odd-value entropy, $S_1 = 1$ all the time. For the other odd-values entropy, S_i $(i = 3, 5, 7, 9)$ increase to a maximal values with the alternation ratio δ decreases. With the alternation ratio δ decreases more, S_i $(i = 3, 5, 7, 9)$ decrease, and they approach to the limit case $\delta = -1$. The entropy S_i $(i = 2, 4, 6)$ increase with the alternation ratio δ decreases. The cases of S_{10} and S_8 are similar to S_i $(i = 3, 5, 7, 9)$. $S_8 = 2.002$ and $S_{10} = 2.003$ when $\delta = -0.3$.

In order to calculate logarithmic behavior of entropy accurately, the number of the alternating spin chain must be large. The method of DMRG without sweeps [20, 21] is applied here. The lattice with opened boundary condition is cleaved into two blocks and one site in the middle as shown in Fig. 2. The super-blocks consists of a system block and an environment block, and one additional site. Two density matrices are computed for each block respectively, and the basis set for each block originates from its corresponding density matrix. Each block increases in size by a single site in central in the DMRG iteration. The total number of density matrix eigenstates held in system block $m_1 = 128$ and in environment block $m_2 = 64$ in the basis truncation procedure. In order to judge the accuracy of our result, The case $\delta = 0$ is used. The corresponding results of finite spin chain, which is predicted by conformal field Theory (CFT) [22], can be considered as benchmark. It can be

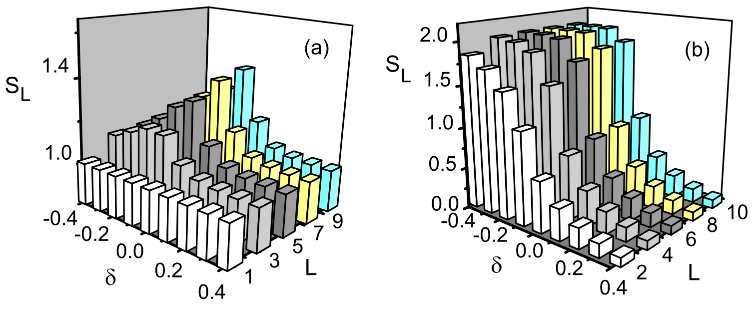

Fig. 1 (**a**) The even-values entropy between subsystem L contiguous qubits and rest of the spin chain are plotted as function of L. (**b**) The odd-values entropy between subsystem L contiguous qubits and rest of the spin chain are plotted as function of L

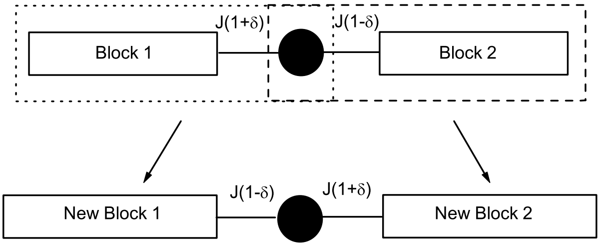

Fig. 2 The augmentation process within one DMRG iteration. Augmentation 1 and augmentation 2 gives the new Block 1 and Block 2 respectively in the next DMRG iteration

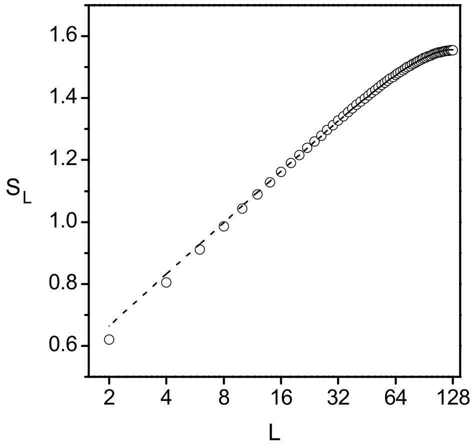

Fig. 3 The linear entropy between contiguous L qubits and remain $N-L$ qubits are plotted as function of subsystem length. *The dash line* is obtained by CFT and ◯ is obtained by DMRG. The length of spin chain $N = 256$ is chosen

written as

$$S_L = \frac{c}{6}\log_2\left[\frac{N}{\pi}\sin\left(\frac{\pi}{N}L\right)\right] + A, \tag{6}$$

where A is a non-university constant [23, 24]. Our programs were performed in personal computer. There are large oscillations between even and odd L-values entropy. To avoid these relatively large oscillations, the even total length is chosen. The results $\delta = 0$ for large L i.e. $L > 8$ agree with result by CFT very well. It is presented in Fig. 3.

In order to know the effect of alternation ratio of exchange interactions on different length spin chain. The entropy is plotted as function of subsystem length for different length of spin chain in Fig. 4. It is shown that the entropy increases with the subsystem increasing when $N = 40$ or $N = 80$. The case is not the same for $N = 256$. For the case $\delta = 0.01$, when $L < 68$, the entropy increase with the subsystem increasing. When $L > 68$, the entropy reach a plateau. The stably value is 1.017. The case is not for $\delta = -0.01$. The Fig. 5 gives ΔS as function of the subsystem for different δ. The ΔS defined as

$$\Delta S_L = S_L(\delta) - S_L(\delta = 0). \tag{7}$$

It is shown that the effect of δ on entropy increase with the subsystem increasing and the all even-values entropy decrease, when $\delta > 0$. While $\delta < 0$, the effect of δ will reach a peak when $L = 52$.

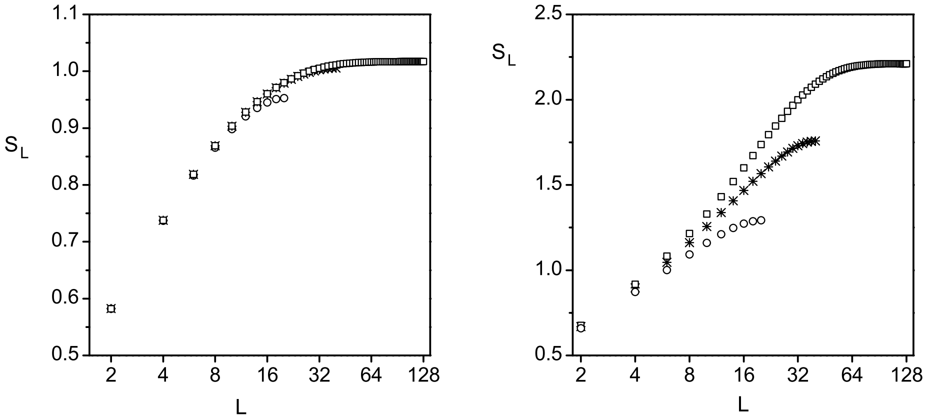

Fig. 4 The entanglement entropy is plotted as function of subsystem length for different length of spin chain. $\delta = 0.01$ is in *right* and $\delta = -0.01$ is in *left*. $N = 40$ (◯), $N = 80$ (⁎), and $N = 256$ (□)

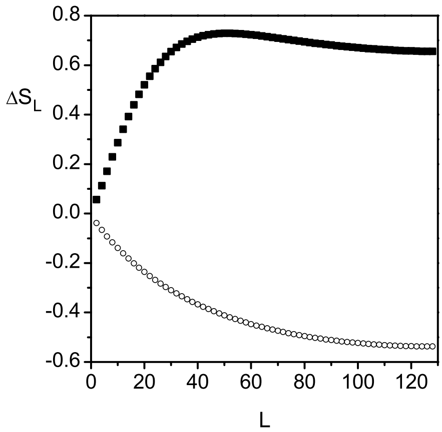

Fig. 5 The entropy ΔS is plotted as function of subsystem length for different δ. $\delta = -0.01$ (◯) and $\delta = 0.01$ (■)

4 Discussion

It is clear that the alternating interaction have a strong influence on the entanglement of the two subsystems [25, 26]. For pairwise entanglement between the first spin and the second spin, the two boundary spins will have a strong tendency to form a singlet pair when the alternating interaction δ is large. This will reduce the entanglement entropy between the boundary of the spin subsystems and the rest of the system. When δ is small, for pairwise entanglement between the second spin and the third spin, the two spins will have a strong tendency to form a singlet pair. This will increase the entanglement between the boundary of the spin subsystems and the rest of the system. It is well-known that the value of entanglement entropy is mainly determined by the density-matrix spectra, extremely by the few largest eigenvalues of the reduced density matrix [10, 11, 21]. The entanglement entropy in the antiferromagnetic Heisenberg chain with alternating interaction is investigated in the paper. Alternating interaction can affect the entropy between two subsystems by changing the distribution of the reduced density-matrix spectra. A feeble alternation has strong effect on the entanglement.

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